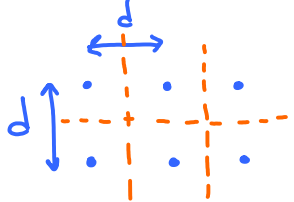


# Quiz 2 Solution

Tuesday, August 20, 2013  
3:38 PM

Find  $P(\epsilon)$  for each of the following constellation  
Assume equiprobable message.  
 $N \sim \mathcal{N}(0, 2^2)$

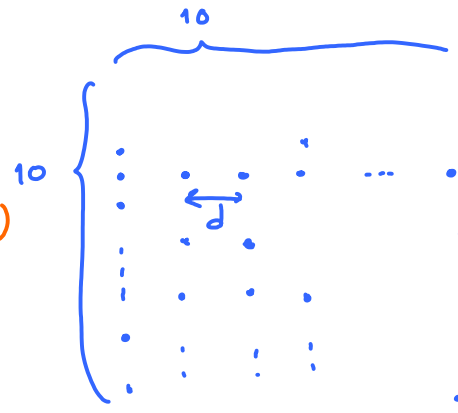
(a) 

$(M=6)$   
 $q = Q\left(\frac{d}{\sqrt{2}\Delta}\right) = Q\left(\frac{4}{2\sqrt{2}}\right)$

$$P(\epsilon) = \frac{1}{6} \left( 4 \times (2q - q^2) + 2 \times (3q - 2q^2) \right)$$

$$= \frac{1}{6} (14q - 8q^2)$$

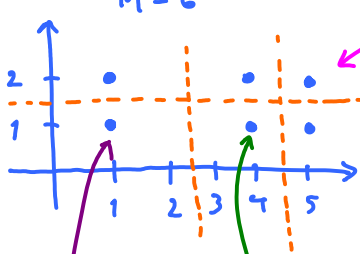
$$= \frac{7}{3} Q(1) - \frac{4}{3} Q^2(1)$$

(b) 

$(M=100)$

$$P(\epsilon) = \frac{1}{100} \left( 4 \times (2q - q^2) + 32(3q - 2q^2) + 64(4q - 4q^2) \right)$$

$$= \frac{1}{100} (360q - 324q^2) = 3.6 Q(1) - 3.24 Q^2(1)$$

(c) 

$M=6$

$P(\epsilon) = 1 - (1 - q_1)(1 - q_1); \quad q_1 = Q\left(\frac{1/2}{2}\right) = Q\left(\frac{1}{4}\right)$

$$= 1 - (1 - 2q_1 + q_1^2)$$

$$= 2q_1 - q_1^2$$

$P(\epsilon) = 1 - (1 - q_1)(1 - q_2) \quad P(\epsilon) = 1 - (1 - q_1 - q_2)(1 - q_1); \quad q_2 = Q\left(\frac{3/2}{2}\right) = Q\left(\frac{3}{4}\right)$

$$= 1 - (1 - q_1 - q_2 + q_1 q_2) = 1 - (1 - q_1 - q_2 - q_1 + q_1^2 + q_1 q_2)$$

$$= q_1 + q_2 - q_1 q_2 = 2q_1 + q_2 - q_1^2 - q_1 q_2$$

$$P(\epsilon) = \frac{1}{3} \begin{pmatrix} 2q_1 & -q_1^2 \\ q_1 & +q_2 - q_1 q_2 \\ 2q_1 & -q_1^2 + q_2 - q_1 q_2 \end{pmatrix} = \frac{1}{3} (5q_1 + 2q_2 - 2q_1^2 - 2q_1 q_2)$$

Alternatively 

$$P(\epsilon) = \frac{1}{3} (q_2 + q_1 + q_2 + q_1)$$

$$= \frac{2}{3} (q_1 + q_2)$$



$$P(\epsilon) = q_1$$

$$P(\epsilon) = 1 - \left(1 - \frac{2}{3}(q_1 + q_2)\right)(1 - q_1)$$

$$\begin{aligned} &= \sqrt{1 - \left(1 - \frac{2}{3} \sigma_1 - \frac{2}{3} \sigma_2 - \sigma_1 + \frac{2}{3} \sigma_1^2 + \frac{2}{3} \sigma_1 \sigma_2\right)} \\ &= \frac{5}{3} \sigma_1 + \frac{2}{3} \sigma_2 - \frac{2}{3} \sigma_1^2 - \frac{2}{3} \sigma_1 \sigma_2 \end{aligned}$$